hsm
HAUSDORFF SCHOOL
FOR ADVANCED STUDIES
IN MATHEMATICS

UNIVERSITÄT BONN

Hausdorff School

"Trending Tools for the Solvability of Nonlocal Elliptic and Parabolic Equatations"

June 28 to July 2, 2021

organized by
Begoña Barrios Barrera, María Medina de la Torre

Abstracts

Eleonara Cinti (Università di Bologna)

Quantitative stability estimates for fractional inequalities

Abstract: We present stability results for some functional inequalities (such as the Faber-Krahn and the isocapacitary inequality) in the nonlocal setting. The proof is based on some ideas by Hansen and Nadirashvili (who considered the classical local case) and uses the so-called Caffarelli-Silvestre extension.

Francesca Da Lio (ETH Zürich)

Analysis of nonlocal conformal invariant variational problems

Abstract: There has been a lot of interest in recent years for the analysis of free-boundary minimal surfaces. In the first part of the course we will recall some facts of conformal invariant problems in 2D and some aspects of the integrability by compensation theory. In the second part we will show how this theory can be extended to the nonlocal framework.

Leandro Del Pezzo (Universidad de Buenos Aires)

Fractional convexity

Abstract: In this talk, we introduce a notion of fractional convexity that extends naturally the usual notion of convexity in the Euclidean space to a fractional setting. With this notion of fractional convexity, we study the fractional convex envelope inside a domain of an exterior datum (the largest possible fractional convex function inside the do- main that is below the datum outside) and show that the fractional convex envelope is characterized as a viscosity solution to a non-local equation that is given by the infimum among all possible directions of the 1dimensional fractional Laplacian. In addition, we find that so- lutions to the equation for the convex envelope are related to solutions to the fractional Monge-Ampere equation. The results of the talk have been obtained in collaboration with J. Rossi (UBA) and A. Quaas (USM).

Monica Musso (University of Bath)

Blow-up solution for the energy-critical heat equation

Abstract: In this course we will discuss some classical results on phenomena of blow-up for solutions of the critical Fujita equations. We will present some results on infinite time blow-up and also on finite-time blow-up successfully obtained in recent years using the inner-outer method. We will explain this method and its possible applications in other contexts.

Benedetta Noris (Politecnico di Milano)

A supercritical elliptic equation in the annulus

Abstract: When searching for solutions to Sobolev-supercritical elliptic problems, a major difficulty is the lack of Sobolev embeddings, that en- trains a lack of compactness. In this talk, I will discuss how symmetry and monotonicity properties can help to overcome this obstacle. In par- ticular, I will present a recent result concerning the existence of a new type of axially symmetric solutions to a semilinear elliptic equation, obtained by a combination of variational and topological techniques. This is a work in collaboration with A. Boscaggin, F. Colasuonno and T. Weth.

Pablo Ochoa (Universidad Nacional de Cuyo-Conicet)

Capacity-based conditions for existence of solutions to fractional elliptic problems with first-order terms

Abstract: In this talk, we will discuss the existence of distributional solutions to fractional elliptic problems with non-linear first-order terms and measure data ! in RN. It is well-known in the literature that solutions to elliptic problems with superlinear growth in the gradient exist if the source is sufficiently small. By appealing to Potential Theory, the size on the source may be given in terms of capacity. In the case of fractional problem, we will see that if the measure data is locally controlled by the Riesz fractional capacity, then there is a global solution for the problem under study. We will also provide a partial converse of the above result under enough global regularity of the solutions. Finally, we will show how to apply the main results to get estimates of the solutions in terms of the data in different function spaces.

Remy Rodiac (Université Paris-Saclay)

Interacting helical travelling waves for the Gross-Pitaevskii

Abstract: In this talk I will explain how to construct some special travelling waves for the Gross-Pitaevskii equation. The vortex set of these travelling waves is a union of helical vortex filaments of degree one placed at the vertices of a regular polygon. Moreover these filaments can be thought of as close to each other and interacting with each other. The energy interaction is analogous to the energy for the logarithmic n-body problem. These travelling waves are constructed via a Lyapounov-Schmidt method. This is a joint work with Juan Davila, Manuel del Pino and Maria Medina.

Xavier Ros-Oton (Univesity of Zurich)

Regularity of free boundaries in obstacle problems

Abstract: Free boundary problems are those described by PDE that exhibit a priori unknown (free) interfaces or boundaries. Such type of problems appear in Physics, Geometry, Probability, Biology, or Finance, and the study of solutions and free boundaries uses methods from PDE, Calculus of Variations, and Geometric Measure Theory. The main mathematical challenge is to understand the regularity of free boundaries.

The Stefan problem and the obstacle problem are the most classical and motivating examples in the study of free boundary problems. The goal of this course is to introduce these free boundary problems, prove some of the main known results in this context, and give an overview of the current research and open problems.

Diana Stan (Universidad de Cantabria)

The fast p-Laplacian evolution equation. Global Harnack principle and fine asymptotic behavior

Abstract: We study fine global properties of nonnegative solutions to the Cauchy Problem for the fast p-Laplacian evolution equation on the whole Euclidean space, in the so-called "good fast diff usion range". It is well-known that non-negative solutions behave for large times as B, the Barenblatt (or fundamental) solution, which has an explicit expression. We prove the so-called Global Harnack Principle (GHP), that is, precise global pointwise upper and lower estimates of nonnegative solutions in terms of B. This can be considered the nonlinear counterpart of the celebrated Gaussian estimates for the linear heat equation. To the best of our knowledge, analogous issues for the linear heat equation, do not possess such clear answers, only partial results are known. Also, we characterize the maximal (hence optimal) class of initial data such that the GHP holds, by means of an integral tail condition, easy to check. Finally, we derive sharp global quantitative upper bounds of the modulus of the gradient of the solution, and, when data are radially decreasing, we show uniform convergence in relative error for the gradients. This is joint work with Matteo Bonforte (UAM-ICMAT, Madrid, Spain) and Nikita Simonov (Ceremade-Univ. Paris-Dauphine, Paris, France).
